

Incorporating taxation in the valuation of variable annuity contracts: the case of the guaranteed minimum accumulation benefit

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Variable Annuities

Variable Annuities (VAs) were first introduced in the early 1950s and various 'GMxBs' have become available:

- Guaranteed Minimum Death Benefit introduced in 1980s.
- Guaranteed Minimum Living Benefits introduced in late 1990s.
 - [GMAB - Accumulation](#),
 - GMIB - Income,
 - GMWB - Withdrawal,
 - (GLWB - Lifelong form of GMWB).

Classical unit-linked maturity products with ongoing fees to fund the guarantee and possibility to surrender have very similar methodological structures!

VA market over the world

- In the US, VA industry is large: US\$1.95 trillion as of first quarter of 2018 (Insured Retirement Institute 2018).
- In Australia and Europe, the market is very thin:
 - In Australia, there are only a few notable players.¹
 - In Europe, the VAs' market was worth 188 billion in 2010 (EIOPA 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped their VA offering.
- In Japan, the VA market grew from a market of less than \$1 billion in 2000 to over \$50 billion, subsequent to a period of financial deregulation in the late 90s (Zhang 2006).

¹e.g. AMP Financial Services, BT Financial Group and MLC. (Vassallo et al. 2016)

Research questions

- What is the impact of tax on surrender behaviour? *Allowing for losses to offset gains is beneficial to policyholders and insurers.*
- How does this impact pricing (from policyholder's and insurer's perspective)? *Policyholder is willing to pay less if losses cannot offset gains.*
- And what if capital losses do offset gains? *policyholder is willing to pay more as losses are also beneficial them at the expense of the government.*
- What is the effect of the market conditions? *the Sharpe ratio in difference drives dramatically the popularity of the GMAB in the market.*

Surrender behavior

- GMABs promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract.
- Typically, the valuation frameworks study the effect of the underlying fund distribution (GBM, Levy, etc) on the fee.
- Recently, the surrender behavior is studied more closely in the literature (Bernard et al. 2014; Kang and Ziveyi 2018) as underpricing lapse risk has resulted in significant losses for insurers (Moody's Investor Service 2013).
- **Here:** the contract can be surrendered at any time prior to maturity, and the payments are liable for **taxes** (policyholder perspective).

Importance of Incorporating Tax

- One of the main attractive features of VAs is their tax-advantaged investing (Milevsky and Panyagometh 2001; Brown and Poterba 2006).
- Incorporating taxation in riders such as GMWB reconciles empirically observed fees with the theory (Moenig and Bauer 2015).
- The financial planning literature has long looked at ways of providing rules to follow so as to maximise post-tax returns (Sumutka et al. 2012; Horan and Robinson 2008).
- **In this study:** we examine the impact of taxation on the optimal surrender behaviour in GMABs and pricing.
- Shade light on why they haven't been popular in markets like Australia or the European Union.

Two tax regimes

- In Moenig and Bauer (2015) the authors study the effect of tax on **capital gains only** [GMWB].
- They note that it reconciles **theoretical fees with those traded in the US market**.
- However, in some tax regimes capital **losses can offset capital gains**, lowering the total tax liability.
- **Here:** we study the policyholder behavior without tax [classical academic assumption], with tax on capital gains only [recent development] and when capital losses can offset gains [*novelty*].

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GMAB product

- Policyholder invests an initial amount x_0 and at maturity receives the greater of the guarantee G and the fund value.
- To finance the guarantee, the insurer charges a continuously compounded fee, q , as a percentage of the fund.
- The income of the policyholder is taxable, however they are not taxed until early surrender or maturity.
- The taxable income of the policyholder at maturity can be re-written as:

$$\overbrace{\max(G, x_T)}^{\text{guarantee}} - \underbrace{(x_0 + C_0)}_{\text{initial payment + upfront cost}} - \overbrace{y(T)}^{\text{total fees}} . \quad (1)$$

- If tax only on capital gains: $[\text{Equation (1)}]_+$,
- If losses offset gains: Equation (1).

Surrender

- The GMAB rider permits the policyholder to surrender the VA anytime prior to maturity.
- Policyholders are not eligible for the guarantee if they surrender early (Kang and Ziveyi 2018).
- Upon surrender, the insurer will pay $\gamma_\nu x_\nu$, where $(1 - \gamma_\nu)$ is the surrender penalty.
- In the event of early surrender at time ν , the taxable income will thus be

$$[\gamma_\nu x_\nu - x_0 - C_0 - y(\nu)]_+ \quad (2)$$

The governing PDE

- Value of the GMAB: $u(x, y, \nu)$ with x fund value, y total fees paid and ν time elapsed since purchase.
- The fund evolves as a Geometric Brownian Motion process.
- If t represents the contract's time to maturity, u will satisfy the PDE:

$$\frac{1}{2}\sigma^2x^2u_{xx} + x \cdot q \cdot u_y + (r - q) \cdot x \cdot u_x - r \cdot u - u_t = 0. \quad (3)$$

- The boundary conditions capture the taxes paid upon surrender or maturity.
 - With tax-gains only
 - With tax-losses offset
- Observe that for sufficiently large fees paid and no offset, the taxable amount is zero \rightarrow no taxation case. No tax
- The boundaries will change when looking at the insurer's perspective. Insurer

Solution methodology

- We use the Method of Lines to solve Equation (3) (Meyer and Van der Hoek 1997).
- The PDE is discretised in t and y , while continuity is maintained in x .
- The differentials u_t and u_y are re-expressed using a finite different approximation, for example

$$u_t = \begin{cases} \frac{u^- - u_{k,n-1}}{\Delta t} & \text{if } n = 1, 2 \\ \frac{3}{2} \frac{u^- - u_{k,n-1}}{\Delta t} - \frac{1}{2} \frac{u_{k,n-1} - u_{k,n-2}}{\Delta t} & \text{if } n \geq 3 \end{cases} \quad (4)$$

- This is a fast and accurate methodology (Meyer and Van der Hoek 1997; Chiarella et al. 2009) and has already proved useful in the VA space (Kang and Ziveyi 2018).

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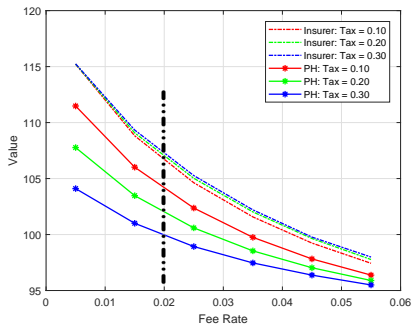
Financial base case parameters

Classical academic assumptions

Parameter	Value	
r	0.03	risk-free rate
σ	0.30	fund volatility
τ	0.225	tax rate
x_0	100	initial premium
G	100	guarantee
T	15	maturity
κ	0.005	penalty

- Weekly time discretisation.
- Maximum possible fund value and total fees set to $4 \cdot G$.

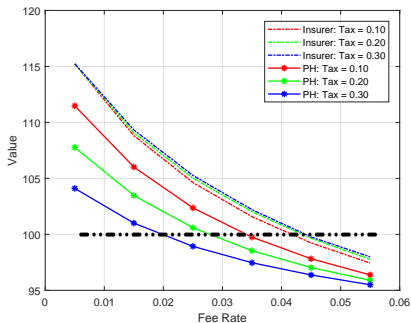
Capital gains (no offset) - Value to PH and insurer



- Policyholder's value **decreases** with tax: for a given fee, all gains are taxed and losses cannot offset them.
- Insurer's value (slightly) **increases** with tax: policyholder behaves as to maximize post-tax value. Higher tax \rightarrow delays surrender and increases fees to the insurer.

Capital gains (no offset) - Value to PH and insurer

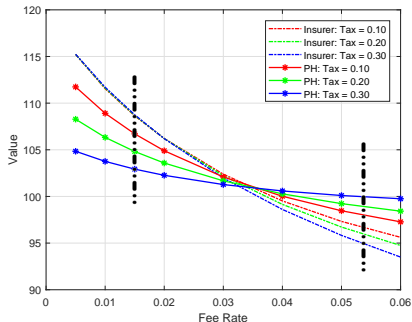
- Insurer's and policyholder's value **decrease** with fees → higher fees incentivize early surrender (loss-loss situation)
⇒ lower policyholder fees for higher tax rates and conversely (slightly) higher insurer fees for higher taxes.
- Policyholder's fee lies below insurer's fee!



- Less interesting to enter the market if potential gains are lower!.
- A market in this tax setting may not exist!.

Surrender boundary

Capital losses can offset gains - Value to PH and insurer



Policyholder:

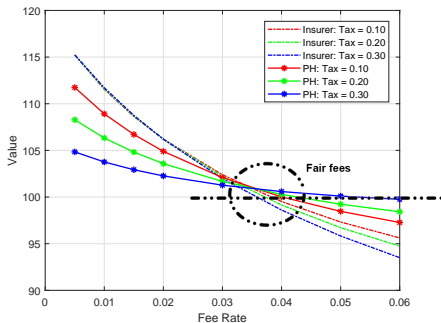
- * if fee < fair fee → higher potential capital gains → value decreases with tax.
- * if fee > fair fee → higher potential losses → value increases with tax (tax back).

Insurer:

- * as tax increases → policyholders are more likely to delay surrender to obtain a certain post-tax value → higher fee income.
- * as fee increases → higher fee income.

Capital losses can offset gains - Value to PH and insurer

- Higher (than fair) fees increase “losses” benefitting policyholders and insurer at the expense of the government! Surrender boundary
- Insurer’s fee lies below the fee the policyholder is willing to pay (compare e.g. blue lines).



The difference between the insurer and policyholder value is the value to the government. The point at which they meet is where the value to the government is zero.

Fair Fees under different tax regimes

Tax regime	$q^{P,*}$ (% p.a.)	$q^{I,*}$ (% p.a.)
No tax	3.91	3.91
$\tau = 0.30$, offset allowed	5.25	3.56
$\tau = 0.30$, no offsets	1.94	4.32

Notes: Surrender boundary no tax

- No tax vs offset: the policyholder is willing to pay more than in the no tax regime as any losses will be beneficial to them. Similarly, the insurer is willing to enter the contract at a lower rate at the expense of the government.
- No tax vs no offset: the policyholder is willing to pay a much lower fee to realise gains \rightarrow higher post-tax value. Similarly, the insurer needs a higher fee to compensate the surrender behavior. A market may not exist under these financial parameters.

Other market assumptions

- Overall, we observe that allowing for losses to offset gains enhances the market.
- The gap between $q^{p,*}$ and $q^{i,*}$ decreases.
- The tax regimes affects the attractiveness of the GMABs.
- What about the financial market assumptions?

Fair fees for other financial market settings

Country	Losses offset		Losses not offset	
	$q^{P,*}$	$q^{I,*}$	$q^{P,*}$	$q^{I,*}$
US	5.94	3.04	2.18	3.12
JP	4.28	3.93	3.45	4.57
AU	1.97	3.42	1.91	3.53
EU	4.82	6.48	4.85	7.10

Notes: Market assumptions for US, JP, AU, EU

- In the US and Japan, market is possible only when losses offset gains ($q^{P,*} > q^{I,*}$).
- Allowing for offset reduces the pricing gap.
- Yet, insufficient to enhance the market in Australia and Europe.

Tax matters, but, is it a deal breaker?

- We observe that tax alters the fair fee, sometimes yielding to lower demand than supply prices.
- However, can the product still be profitable?
- Yes, we find that² when charging the demand price, insurance companies would be profitable on average³.
- We also show how investment policy, as reflected in the Sharpe ratio, impacts and interacts with policyholder persistency.

²The profit and loss (P&L) tables provide an overview of the surrender fee that the insurer receives upon early surrender, the cost of providing the guarantee, management fees required to fund the insurance product, average time elapsed in the contract before surrender (if any).

³The net profit is calculated as the management fees, complemented by the surrender fee reduced by the guarantee cost. We also show the net profit quantiles in order to inform about their skewness and level

Table: Profit and Loss profiles for base case. Here SR denotes the Sharpe ratio, and 'mgmt fees' denotes the management fees in basis fees. q^p is the fair fee implied by the parameter set considered in each block.

(BASE Parameters)	tax free ($q^p = 3.32\%$)	no offset ($q^p = 1.92\%$)	offset ($q^p = 1.97\%$)
Surrender fee	0.3831	2.3287	1.8858
Guarantee cost	0	0	0
Mgmt fees	59.245	30.5879	32.9456
Surrender Rate	100%	100%	100%
Avg time elapsed	14.497	12.2795	12.8382
Net profit	59.63	32.91	34.8314
Net Profit Qtiles	58.9,59.3,59.6	32.6,33.0,33.3	34.5,34.9,35.2

- In all cases the policyholder surrenders, but without offset happens more often (and yields more surrender fees)
- Lower demand fees yield lower management fees, but overall all simulations yield profits.
- Quartiles show that there is high certainty of this net profit and that it is not too skewed.

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Conclusion

- Tax is a key aspect of financial planning and insurance take-up.
- We illustrate the impact of various tax systems, including the realistic case when losses can offset gains.
- The relationship between fees, behavior and contract value vary across systems: e.g., when losses offset gains then the contract is interesting for both parties at the expense of the government.
- The method of lines used enables us to efficiently determine optimal surrender boundaries, contract values and fair fees.
- *Next steps*: adding withdrawals or regular premiums?

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Thanks

Thank you for your attention
Questions?

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Boundary conditions with tax (losses don't offset gains)

In order to obtain the contract value from the policyholder perspective, we solve equation (3) subject to the following boundary conditions:

$$u(x, y, 0) = \max(x, G) - \tau [\max(x, G) - y - x_0 - C_0]_+, \quad (5)$$

$$u(s(t, y), y, t) = s(t, y)\gamma_t - \tau [s(t, y)\gamma_t - y - x_0 - C_0]_+, \quad (6)$$

$$u(0, y, t) = (G - \tau [G - y - x_0 - C_0]_+)e^{-rt}, \quad (7)$$

$$u_x(s(t, y), y, t) = \gamma_t - \tau \gamma_t \mathbb{I}\{s(t, y)\gamma_t - y - x_0 - C_0 > 0\}, \quad (8)$$

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Boundary conditions with tax (losses offset gains)

We explore the case in which the capital losses on the GMAB product can be used to offset other income sources, as is the case for nonqualified plans in the US. Mathematically, this entails the following replacement:

$$\tau(\gamma_t x - y - x_0)_+ \rightarrow \tau(\gamma_t x - y - x_0)$$

in equation the boundary conditions (5),(6), (7) and (8). Therefore, the new problem requires us to solve the PDE (3) subject to the boundary conditions:

$$u(x, y, 0) = \max(x, G) - \tau(\max(x, G) - y - x_0 - C_0), \quad (9)$$

$$u(s(t, y), y, t) = s(t, y)\gamma_t - \tau(s(t, y)\gamma_t - y - x_0 - C_0), \quad (10)$$

$$u(0, y, t) = [G - \tau(G - y - x_0 - C_0)]e^{-rt}, \quad (11)$$

$$u_x(s(t, y), y, t) = \gamma_t - \tau\gamma_t. \quad (12)$$

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Boundary conditions without tax

Putting $u_y = 0$ into equation (3), we recover the following 2 dimensional PDE

$$\frac{1}{2}\sigma^2x^2u_{xx} + (r - q) \cdot xu_x - ru - u_t = 0. \quad (13)$$

which must be solved subject to the following boundary conditions:

$$u(x, Y, 0) = \max(x, G), \quad (14)$$

$$u(s(t, Y), Y, t) = s(t, Y)\gamma_t, \quad (15)$$

$$u(0, Y, t) = Ge^{-rt}, \quad (16)$$

$$u_x(s(t, Y), Y, t) = \gamma_t. \quad (17)$$

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Boundary conditions for the insurer's liabilities

To obtain the value of the contract from the insurer's perspective, henceforth to be referred to as the insurer's liabilities, the partial differential equation (3) must be solved subject to boundary conditions which reflect the total before tax payments the insurer must make to the policyholder:

$$u_{Ins}(x, y, 0) = \max(x, G) \quad (18)$$

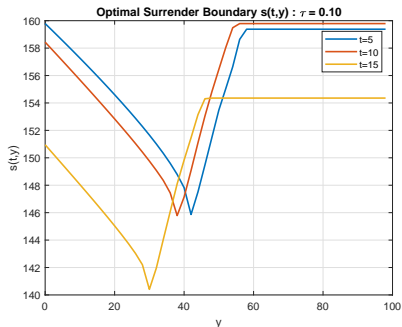
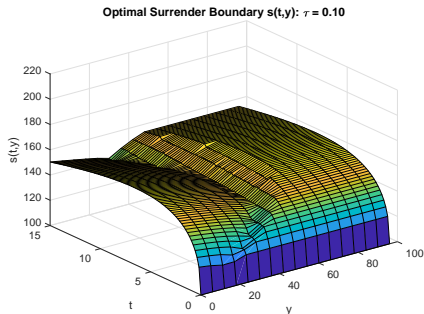
$$u_{Ins}(s(t, y), y, t) = s(t, y)\gamma_t \quad (19)$$

$$u_{Ins}(0, y, t) = Ge^{-rt} \quad (20)$$

$$\frac{\partial u_{Ins}(s(t, y), y, t)}{\partial x} = \gamma_t \quad (21)$$

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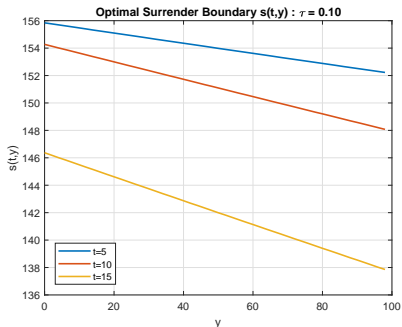
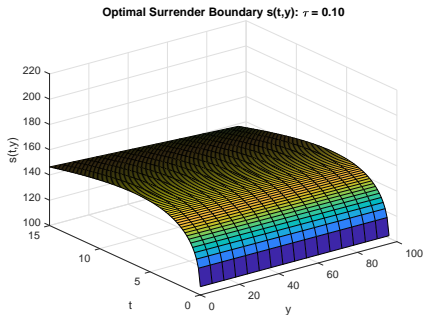
Surrender boundary **no** offset ($\tau = 0.10$)



The 'valley of surrender', a combination of values of cumulative fees paid y and time to maturity t is driven by the fact that capital losses cannot be claimed on the product.

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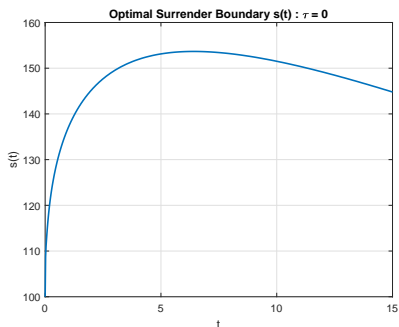
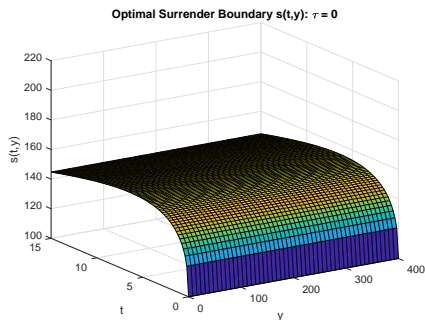
Surrender boundary **with** offset ($\tau = 0.10$)



The surrender surface $s(t,y)$ is monotonically decreasing in y . This is because all else equal, having already paid a greater sum of fees will reduce taxable income.

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Surrender surface when $\tau = 0$



The surrender boundary is independent of the cumulative fees paid y because there is no tax. The shape agrees with those presented by Bernard et al. (2014).
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Market assumptions for US, JP, AU and EU

Parameter	r (%)	σ (%)	τ (%)	x_0	G	T	κ
US	3.00	19	15.0	100	125	15	0.005
JP	1.20	24	20.0	100	100 ⁴	15	0.005
AU	3.00	20	22.5	100	125	15	0.005
EU	3.40	31	20.0	100	125	15	0.005

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⁴The guarantee 125 is not compatible with the low risk-free rate.